Complex Inversion: $z \rightarrow \frac{1}{z}$

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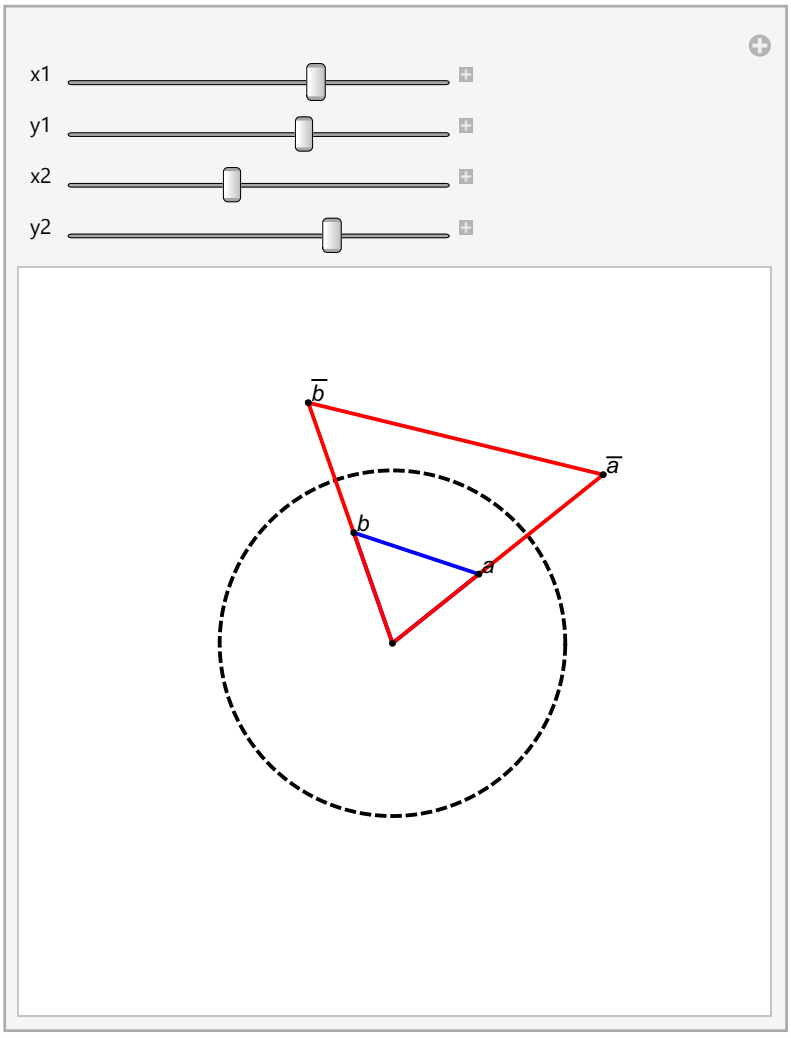
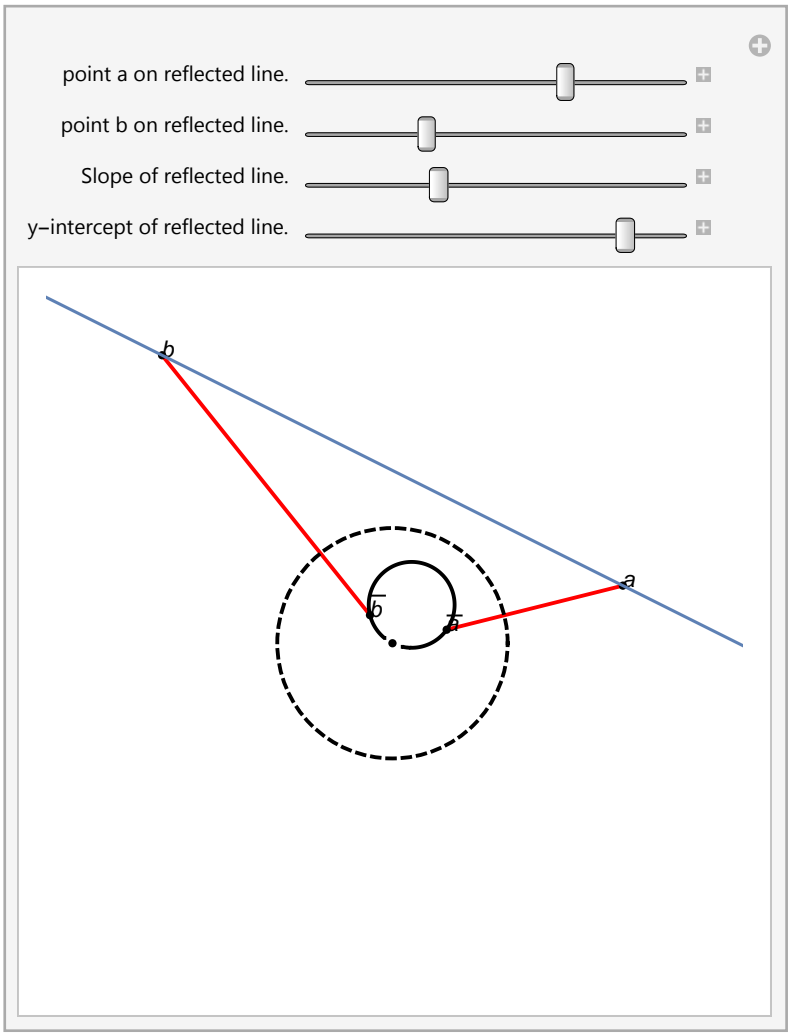
Inversion of points a and b.

```
R = 1;(*radius of inversion circle*)
plotRange =
  Max[EuclideanDistance[invPt1, {0, 0}], EuclideanDistance[invPt2, {0, 0}]];

Ik[x_, y_, q1_, q2_, R_] := ((q1 + I * q2) * Conjugate[x + I * y] + (R^2 - Abs[q1 + I * q2]^2)) /
    (Conjugate[x + I * y] - Conjugate[q1 + I * q2]);(*Inversion map under circle k*)


Manipulate[

 pt0 = {0, 0};
 pt1 = {x1, y1};
 pt2 = {x2, y2};
 (*The inversion map is complex so I have to
  separate real and imaginary parts for plotting in R^2*)
 invPt1 = {Re[Ik[x1, y1, 0, 0, 1]], Im[Ik[x1, y1, 0, 0, 1]]};
 invPt2 = {Re[Ik[x2, y2, 0, 0, 1]], Im[Ik[x2, y2, 0, 0, 1]]};

 triangle1 = Triangle[{pt0, pt1, pt2}];
 triangle2 = Triangle[{pt0, invPt1, invPt2}];


 plotRange =
  Max[EuclideanDistance[invPt1, {0, 0}], EuclideanDistance[invPt2, {0, 0}]];
 Graphics[{Dashed, Thick, Circle[pt0, R], EdgeForm[{Blue, Thick}], FaceForm[],
   triangle1, EdgeForm[{Thick, Red}], triangle2, PointSize[0.01],
   Point[{pt0, pt1, pt2, invPt1, invPt2}], Text[Style[a, Medium], pt1 + 0.055],
   Text[Style[b, Medium], pt2 + 0.055], Text[Style[a̅, Medium], invPt1 + 0.055],
   Text[Style[b̅, Medium], invPt2 + 0.055]}, PlotRange → {{-2, 2}, {-2, 2}}]

 , {{x1, 0.5}, -1.5, 1.5}, {{y1, 0.4}, -1.5, 1.5},
 {{x2, -0.224}, -1.5, 1.5}, {{y2, 0.406}, -1.5, 1.5}]
```


x1
y1
x2
y2
$\overline{b}$
$\overline{a}$
b
a

Inversion of points on a line.

```
R = 1;
plotRange =
  Max[EuclideanDistance[invPt1, {0, 0}], EuclideanDistance[invPt2, {0, 0}]];
Ik[x_, y_, q1_, q2_, R_] := ((q1 + I * q2) * Conjugate[x + I * y] + (R^2 - Abs[q1 + I * q2]^2)) /
    (Conjugate[x + I * y] - Conjugate[q1 + I * q2]);
(*Inversion map under circle k*)
m = -0.5;
y = 1.5;
reflectedLine = {t, m * t + y};

t = -1.7;
s = 0.6;
Manipulate[

 (*slope of reflected line*)
 (*y-intercept of reflected line*)

 pt0 = {0, 0};
 pt1 = {t, m * t + y};
 pt2 = {s, m * s + y};
 (*The inversion map is complex so I have to
  separate real and imaginary parts for plotting in R^2*)
 invPt1 = {Re[Ik[t, m * t + y, 0, 0, 1]], Im[Ik[t, m * t + y, 0, 0, 1]]};
 invPt2 = {Re[Ik[s, m * s + y, 0, 0, 1]], Im[Ik[s, m * s + y, 0, 0, 1]]};

 line1 = Line[{pt1, invPt1}];
 line2 = Line[{pt2, invPt2}];

 Show[Graphics[{{Red, Thick, line1, line2}, Dashed, Thick, Circle[pt0, R],
    PointSize[0.012], Point[{pt0, pt1, pt2, invPt1, invPt2}], Thick,
    Text[Style[a, Medium], pt1 + 0.055], Text[Style[b, Medium], pt2 + 0.055],
    Text[Style[a̅, Medium], invPt1 + 0.055], Text[Style[b̅, Medium], invPt2 + 0.055]},
   PlotRange → {{-3, 3}, {-3, 3}}], ParametricPlot[{s, m * s + y}, {s, -5, 5}],
  ParametricPlot[{Re[Ik[t, m * t + y, 0, 0, 1]], Im[Ik[t, m * t + y, 0, 0, 1]]},
   {t, -10, 10}, PlotStyle → {Black, Thick}]]


 , {{t, 2, "point a on reflected line."}, -5, 5},
 {{s, -2, "point b on reflected line."}, -5, 5},
 {{m, -0.5, "Slope of reflected line."}, -1.5, 1.5},
 {{y, 1.5, "y-intercept of reflected line."}, -2, 2}]
```


point a on reflected line.
point b on reflected line.
Slope of reflected line.
y–intercept of reflected line.
b
a
b̄
ā

Inversion of points on a circle.

```
R = 1;
plotRange =
  Max[EuclideanDistance[invPt1, {0, 0}], EuclideanDistance[invPt2, {0, 0}]];
Ik[x_, y_, q1_, q2_, R_] := ((q1 + I * q2) * Conjugate[x + I * y] + (R^2 - Abs[q1 + I * q2]^2)) /
    (Conjugate[x + I * y] - Conjugate[q1 + I * q2]);
(*Inversion map under circle k*)
Manipulate[

 pt0 = {0, 0};
 (*defining points on a circle of radius r and center (c1,c2)*)
 pt1 = r * {Cos[t], Sin[t]} + {c1, c2};
 pt2 = r * {Cos[s], Sin[s]} + {c1, c2};
 (*The inversion map is complex so I must
  separate real and imaginary parts for plotting in R^2*)
 invPt1 = {Re[Ik[r * Cos[t] + c1, r * Sin[t] + c2, 0, 0, 1]],
   Im[Ik[r * Cos[t] + c1, r * Sin[t] + c2, 0, 0, 1]]};
 invPt2 = {Re[Ik[r * Cos[s] + c1, r * Sin[s] + c2, 0, 0, 1]],
   Im[Ik[r * Cos[s] + c1, r * Sin[s] + c2, 0, 0, 1]]};

 line1 = Line[{pt1, invPt1}];
 line2 = Line[{pt2, invPt2}];

 Show[Graphics[{{Red, Thick, line1, line2}, Dashed, Thick, Circle[pt0, R],
    PointSize[0.012], Point[{pt0, pt1, pt2, invPt1, invPt2}], Thick,
    Text[Style[a, Medium], pt1 + 0.055], Text[Style[b, Medium], pt2 + 0.055],
    Text[Style[Overscript[a, _], Medium], invPt1 + 0.055], Text[Style[Overscript[b, _], Medium], invPt2 + 0.055]},
   PlotRange → {{-3, 3}, {-3, 3}}], ParametricPlot[
   {r * Cos[s] + c1, r * Sin[s] + c2}, {s, -5, 5}, PlotStyle → {Green}],
  ParametricPlot[{Re[Ik[r * Cos[t] + c1, r * Sin[t] + c2, 0, 0, 1]],
    Im[Ik[r * Cos[t] + c1, r * Sin[t] + c2, 0, 0, 1]]},
   {t, -10, 10}, PlotStyle → {Blue, Thick}]]


 , {{t, 0, "point a on reflected circle."}, 0, 2 * π},
 {{s, π, "point b on reflected circle"}, 0, 2 * π},
 {{r, 1/5, "radius of reflected circle"}, 0, 1},
 {{c1, 0.5, "x-coordinate of center of reflected circle"}, -1, 1},
 {{c2, 0.5, "y-coordinate of center of reflected circle"}, -1, 1}
]
```

point a on reflected circle.

point b on reflected circle

radius of reflected circle

x–coordinate of center of reflected circle

y–coordinate of center of reflected circle