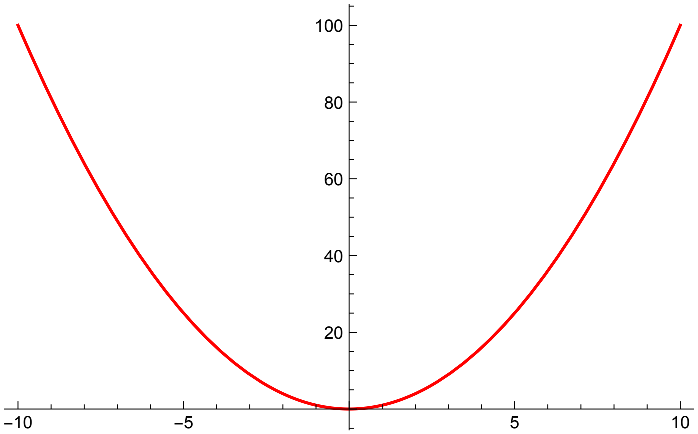
# Fourier Series and Epicycles

Choose a function, g(x), a length and an order for the Fourier Series.

In[9]:=

```
Clear["Global`*"]
L = 4;(*Length of Interval*)
order = 10;(*Order of Fourier Series*)
g[x_] := x^2;(*Function user wants analyzed*)
functionPlot = Plot[g[x], {x, -10, 10}, PlotStyle → {Red}]
f[x_] := x + I * g[x];
(*The program considers the fourier series of this function,
where g(x) is described like a parametric in the complex plane*)
fourierSeries[x_] :=
  Sum[1/(2 * π) * Integrate[f[x] * (Cos[x * t] - I * Sin[x * t]), {x, -π, π}] * E^(I*t*x),
   {t, -order, order}];
Print["The order " , order , " Fourier series of the chosen function: "
  fourierSeries[x]]
coeffsAndAngList = SortBy[
   Table[{1/(2 * π) * Integrate[f[x] * (Cos[x * t] - I * Sin[x * t]), {x, -π, π}], t},
    {t, -order, order}], Abs[#[[1]]] &];
(*A list with the radii (=coefficients) in the first entry of each position,
and the angular velocities in the second. This list is ordered by
 the absolute size of the radii, from smallest to greatest. This
 is so the epicycles will print from smallest to greatest*)

radii = Table[coeffsAndAngList[[i, 1]], {i, 1, Length[coeffsAndAngList]}];
angularVel = Table[coeffsAndAngList[[i, 2]], {i, 1, Length[coeffsAndAngList]}];
```

Out[13]=




The order 10

Fourier series of the chosen function: $\left(-i\,e^{-ix}-3\,i\,e^{ix}+i\,e^{2ix}+\frac{1}{9}\,i\,e^{-3ix}-\frac{5}{9}\,i\,e^{3ix}-\frac{1}{8}\,i\,e^{-4ix}+\frac{3}{8}\,i\,e^{4ix}+\frac{3}{25}\,i\,e^{-5ix}-\frac{7}{25}\,i\,e^{5ix}-\frac{1}{9}\,i\,e^{-6ix}+\frac{2}{9}\,i\,e^{6ix}+\frac{5}{49}\,i\,e^{-7ix}-\frac{9}{49}\,i\,e^{7ix}-\frac{3}{32}\,i\,e^{-8ix}+\frac{5}{32}\,i\,e^{8ix}+\frac{7}{81}\,i\,e^{-9ix}-\frac{11}{81}\,i\,e^{9ix}-\frac{2}{25}\,i\,e^{-10ix}+\frac{3}{25}\,i\,e^{10ix}+\frac{i\,\pi^2}{3}\right)$

## Visualizations.

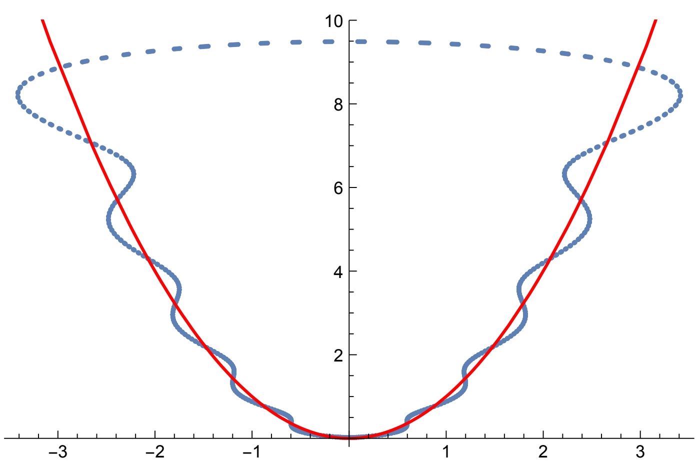
In[20]:=

```
n = Length[coeffsAndAngList];


circles = Table[radii[[i]] * E^((angularVel[[i]])*I*t), {i, 1, n}];
(*This gives a list of all the circles, from smallest to greatest*)
centerCoords = Table[{N[Re[circles[[i]]]], N[Im[circles[[i]]]]}, {i, 1, n}];
(*This breaks the circles into the real and
 imaginary parts to get the coordinates of their centers*)
epicycles = Table[Sum[centerCoords[[j]], {j, i + 1, n}], {i, 1, n}];
(*This places each circle at the edge of the next,
from smallest to greatest. *)
tracingPoint = Sum[centerCoords[[i]], {i, 1, n}];
(*This is the point at the very edge of the series of epicycles
 that will trace the path of the epicycles as they rotate*)

(*---------------------------------------*)

circlesForGraphic = Table[Circle[epicycles[[i]], Abs[radii[[i]]]], {i, 1, n - 1}];
(*This creates the circles that will be used in Graphics*)
AppendTo[circlesForGraphic, Circle[{0, 0}, Abs[radii[[n]]]]];
(*The sum in "epicycles" misses the first circle,
which needs to be drawn as well*)
```

```
linesForGraphic = Table[
   Line[{circlesForGraphic[[i, 1]], circlesForGraphic[[i + 1, 1]]}], {i, 1, n - 1}];



epicyclesGraphic = Graphics[{PointSize[0.025], Purple, Thick,
    Point[{0, 0}], Point[tracingPoint], linesForGraphic, circlesForGraphic},
   PlotRange → N[Sum[Abs[radii[[i]]], {i, 1, n}]]];(*The graphic*)



(*---------------------------------------*)
(*This loop calculates "traceCount" number of
 points along the path of the tracingPoint to be listplot*)
traceCount = 200;
data = {};
For[t = 1, t ≤ traceCount, t = t + 0.1,

 AppendTo[data, tracingPoint];

]

Print["The following list plot traces 200 points along the path of the
   outermost epicycle and juxtaposes it to the plot of the function"]
Show[ListPlot[data], functionPlot]
shape = ListCurvePathPlot[data, PlotTheme -> "Detailed"];
Dynamic[Show[epicyclesGraphic, shape]]
{Slider[Dynamic[t], {0, 10}], Dynamic[t]}
```

```
The following list plot traces 200 points along the path of
  the outermost epicycle and juxtaposes it to the plot of the function
```

Out[33]=



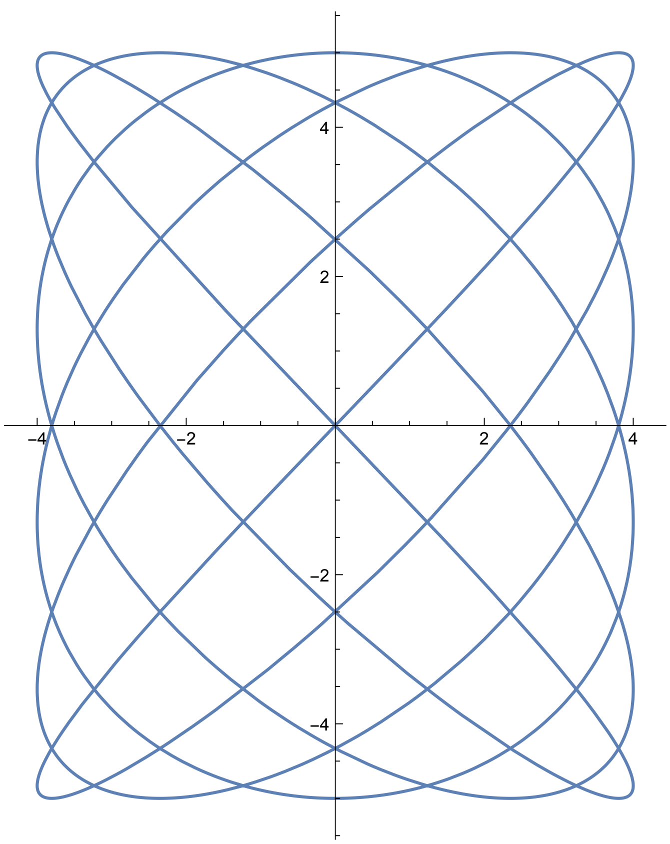
Out[36]= {, 3.00336}

# Making a Lissajous Curve.

In[37]:=

```
Clear["Global`*"]
(*Lissajous Curves of the form (A*Sin(a*t),B*Sin(b*t)), a,b ∈ Z,
have period 2*π, so they can be perfectly approximated by a a series
 of epicycles and provide a wealth of examples to play with.*)
A = 4;(*Build a Lissajous curve*)
B = 5;
a = 6;
b = 5;
lissajousCurve = {A * Sin[a * t], B * Sin[b * t]};
curvePeriod = (2 * π) / GCD[a, b];(*The curve period for a Lissajous curve*)
lissajousPlot = ParametricPlot[lissajousCurve, {t, 0, curvePeriod}]
```

Out[44]=



# Building the Epicycles for the Lissajous Curve.

In[45]:=

```
L = 6; (*Length of Interval*)
order = 10; (*Order of Fourier Series*)
g[x_] := x^2; (*Function user wants analyzed*)
f[t_] := A * Sin[a * x] + I * (B * Sin[b * x]);
(*The program considers the fourier series of this function,
where g(x) is described like a parametric in the complex plane*)
fourierSeries[x_] :=
  Sum[1/(2 * π) * Integrate[f[x] * (Cos[x * t] - I * Sin[x * t]), {x, -π, π}] * E^(I*t*x),
   {t, -order, order}];
Print["The order " , order , " Fourier series of the chosen Lissajous curve: "
  fourierSeries[x]]
coeffsAndAngList = SortBy[
   Table[{1/(2 * π) * Integrate[f[x] * (Cos[x * t] - I * Sin[x * t]), {x, -π, π}], t},
    {t, -order, order}], Abs[#[[1]]] &];
(*A list with the radii (=coefficients) in the first entry of each position,
and the angular velocities in the second. This list is ordered by
 the absolute size of the radii, from smallest to greatest. This
 is so the epicycles will print from smallest to greatest*)

radii = Table[coeffsAndAngList[[i, 1]], {i, 1, Length[coeffsAndAngList]}];
angularVel = Table[coeffsAndAngList[[i, 2]], {i, 1, Length[coeffsAndAngList]}];
```

The order 10

Fourier series of the chosen Lissajous curve: $\left(-\frac{5}{2} e^{-5 i x} + \frac{5}{2} e^{5 i x} + 2 i e^{-6 i x} - 2 i e^{6 i x}\right)$

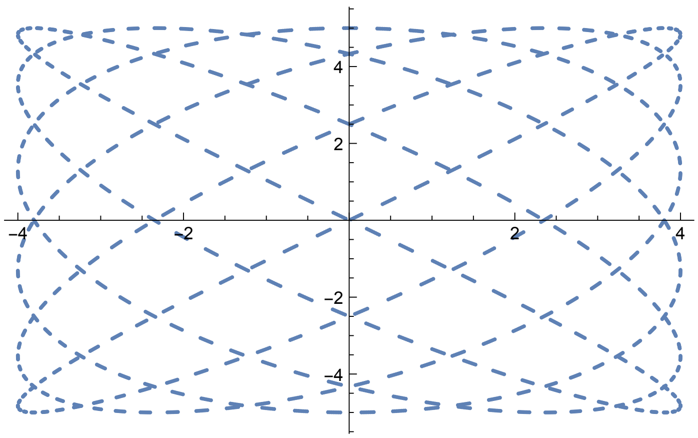
# Visualizations.

In[54]:=

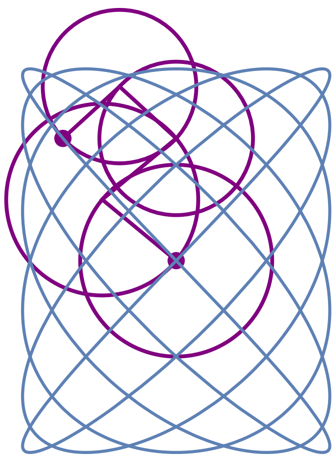
```
n = Length[coeffsAndAngList];

circles = Table[radii[[i]] * E^((angularVel[[i]])*I*t), {i, 1, n}];
(*This gives a list of all the circles, from smallest to greatest*)
centerCoords = Table[{N[Re[circles[[i]]]], N[Im[circles[[i]]]]}, {i, 1, n}];
(*This breaks the circles into the real and
 imaginary parts to get the coordinates of their centers*)
epicycles = Table[Sum[centerCoords[[j]], {j, i + 1, n}], {i, 1, n}];
(*This places each circle at the edge of the next,
```

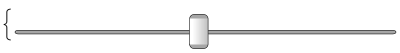
```
from smallest to greatest. *)
tracingPoint = Sum[centerCoords[[i]], {i, 1, n}];
(*This is the point at the very edge of the series of epicycles
 that will trace the path of the epicycles as they rotate*)

(*--------------------------------------*)

circlesForGraphic = Table[Circle[epicycles[[i]], Abs[radii[[i]]]], {i, 1, n - 1}];
(*This creates the circles that will be used in Graphics*)
AppendTo[circlesForGraphic, Circle[{0, 0}, Abs[radii[[n]]]]];
(*The sum in "epicycles" misses the first circle,
which needs to be drawn as well*)
linesForGraphic = Table[
   Line[{circlesForGraphic[[i, 1]], circlesForGraphic[[i + 1, 1]]}], {i, 1, n - 1}];



epicyclesGraphic = Graphics[{PointSize[0.025], Purple, Thick,
    Point[{0, 0}], Point[tracingPoint], linesForGraphic, circlesForGraphic},
   PlotRange → N[Sum[Abs[radii[[i]]], {i, 1, n}]]];(*The graphic*)

(*--------------------------------------*)
(*This loop calculates "traceCount" number of
 points along the path of the tracingPoint to be listplot*)
traceCount = 300;
data = {};
For[t = 1, t ≤ traceCount, t = t + 0.1,

 AppendTo[data, tracingPoint];

]

ListPlot[data]
Dynamic[Show[epicyclesGraphic, lissajousPlot]]
{Slider[Dynamic[t], {0, curvePeriod}], Dynamic[t]}
```

Out[66]=




Out[67]=



Out[68]= {, 3.00336}